

An Updated Precision Estimate of the Hubble Constant and the Age and Density of the Universe in the Decaying Neutrino Theory

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ABSTRACT

We here update the derivation of precise values for the Hubble constant H_0 , the age t_0 and the density parameter Ωh^2 of the universe in the decaying neutrino theory for the ionisation of the interstellar medium (Sciama 1990 a, 1993). Using recent measurements of the temperature of the cosmic microwave background, of the abundances of D, He⁴ and Li⁷, and of the intergalactic hydrogen-ionising photon flux at zero red shift, we obtain for the density parameter of the universe $\Omega h^2 = 0.300 \pm 0.003$. Observed limits on H_0 and t_0 then imply that, for a zero cosmological constant, $H_0 = 52.5 \pm 2.5$ km. sec⁻¹ Mpc⁻¹, $t_0 = 12.7 \pm 0.7$ Gyr and $\Omega = 1.1 \pm 0.1$. If $\Omega = 1$ exactly, then $H_0 = 54.8 \pm 0.3$ km. sec⁻¹ Mpc⁻¹, and $t_0 = 11.96 \pm 0.06$ Gyr. These precise predictions of the decaying neutrino theory are compatible with current observational estimates of these quantities.

Key words: cosmology: dark matter – distance scale

1 INTRODUCTION

Recent developments in attempts to measure the Hubble constant H_0 and the age of the universe t_0 suggest that it would be worth updating the precision values for these quantities derived (Sciama 1990 b) in the decaying neutrino theory for the ionisation of the interstellar medium (Sciama 1990a, 1993). Such updating can take advantage of recent reductions in the uncertainties of each of the quantities which enter into this derivation. These quantities are n_ν/n_γ , n_γ , n_b and m_ν , where n_ν and m_ν are the present cosmological number density and rest mass of the decaying neutrinos, while γ and b refer to cosmic microwave photons and baryons respectively. The current uncertainties in each of the first three quantities enter into the derived value of Ωh^2 at a level of less than 1 per cent, where Ω is the density of the universe in units of the critical density and $H_0 = 100h$ km. sec⁻¹ Mpc⁻¹. The same is true for m_ν in the decaying neutrino theory. Accordingly in this discussion we shall work to four significant figures where necessary.

2 UPDATED INPUT QUANTITIES

(i) n_ν/n_γ

The standard value of this quantity is strongly influenced by the fact that in the early universe $e^- - e^+$ pairs annihilated permanently only after neutrinos had already decoupled from the primordial heat bath. The resulting annihilation photons (or equivalently their entropy) then boosted

the photons in the heat bath but not the decoupled neutrinos. One then finds (Alpher et al 1953) that

$$n_\nu = \frac{3}{11} n_\gamma$$

for each of the three neutrino types ν_e, ν_μ and ν_τ . The decaying neutrino must be either ν_μ or ν_τ , since m_{ν_e} is known to be too low for decay photons from ν_e to be able to ionise hydrogen. We now need to know how accurate is the factor 3/11. This question has been answered by a number of recent calculations which use the Boltzmann equation to take into account the incompleteness of the decoupling of the neutrinos from the heat bath when the electron pairs annihilated permanently. We shall use the detailed calculations of Hannestad and Madsen (1995) who found that, for ν_μ or ν_τ , n_ν is increased by 0.25 percent. We will take this correction into account, and will assume that any further correction is negligible for our purposes.

(ii) n_γ

We know from COBE that the cosmic microwave background has an accurately thermal spectrum; observed RMS deviations are less than 50 parts per million of the peak brightness (Fixsen et al 1996). For such a spectrum of temperature T

$$n_\gamma = 16\pi\zeta(3) (kT/hc)^3, \quad (1)$$

where $\zeta(3) = 1.202$. The most accurate value for T has

been given by Fixsen et al (1996), namely

$$T = 2.728 \pm 0.004 K . \quad (2)$$

Thus the present uncertainty in T is only about 1/7 per cent. We therefore use (1) to calculate n_γ with T given by (2), so that n_γ has an uncertainty of about 3/7 per cent. We find that

$$n_\gamma = 412 \pm 2 \text{cm}^{-3} .$$

Hence

$$\frac{3}{11} n_\gamma = 112.4 \pm 0.5 \text{cm}^{-3} .$$

When we correct for the partial coupling of the neutrinos we obtain

$$n_\nu = 112.6 \pm 0.5 \text{cm}^{-3} .$$

(iii) n_b

We use a value for n_b derived from recent measurements of the abundances of D, He^4 and Li^7 , and the theory of big bang nucleosynthesis. For D we refer to Hata et al (1997) and Songaila et al (1997) who discuss two competing derived values of $\Omega_b h^2$: a low value of $(7.5 \pm 2.5) \times 10^{-3}$ and a high value of $(25 \pm 5) \times 10^{-3}$. However, as Hata et al point out, the high value is inconsistent with the value of $\Omega_b h^2$ derived from recent measured abundances of He^4 and Li^7 , and also with the known number of neutrino types. We therefore adopt the low value of $\Omega_b h^2 = (7.5 \pm 2.5) \times 10^{-3}$ from the deuterium measurements. For the abundance of He^4 we refer to Izotov et al (1997) and Olive et al (1996). From their discussion we adopt a derived value for $\Omega_b h^2$ of $(7 \pm 1) \times 10^{-3}$ from the He^4 measurements. Finally we consider the abundance of Li^7 . For a given observed abundance the theory yields two alternative possible values of $\Omega_b h^2$. Following the recent work of Bonifacio and Molaro (1997) we take for these two alternatives $6.2^{+1.8}_{-1.1} \times 10^{-3}$ and $14.6^{+2.9}_{-3.3} \times 10^{-3}$. In order to be consistent with the D and He^4 values of $\Omega_b h^2$ we adopt the value $6.2^{+1.8}_{-1.1} \times 10^{-3}$. When we combine the results derived from the abundances of the three isotopes, we find consistency if we adopt

$$\Omega_b h^2 = (7 \pm 1) \times 10^{-3} .$$

Compared to our final value for Ωh^2 of 0.3 (see below) we note that the uncertainty in our adopted value for $\Omega_b h^2$ contributes a relative uncertainty of only 0.3 per cent to the final result.

(iv) m_ν

We first relate m_ν to $\Omega_\nu h^2$. From general relativity we have that $\frac{8\pi}{3} G \rho_{crit} = H_0^2$, and so

$$\rho_{crit} = 1.880 \times 10^{-29} h^2 \text{gm cm}^{-3} .$$

We also have from (ii) that

$$\rho_\nu = (112.6 \pm 0.5) m_\nu \text{ mass units cm}^{-3} .$$

Since a mass unit of 1 eV corresponds to 1.783×10^{-33} gm, we find that

$$m_\nu = (93.6 \pm 0.4) \Omega_\nu h^2 \text{ev} .$$

To determine m_ν we follow Sciama (1990b) and use the observed upper limit on the extragalactic hydrogen-ionising flux F at zero red shift to derive an upper limit on the energy E_γ of a decay photon in the rest frame of the decaying neutrino. We then assume that the mass m_2 of the residual neutrino in the decay is much less than m_ν so that we can write $m_\nu = 2E_\gamma$. To keep the uncertainty resulting from this step well below 1 per cent, we explicitly assume that $m_\nu/m_2 > 30$ (as would be easily satisfied, for example, in the see-saw model for neutrino masses (Yanagida 1978, Gell-Maun et al 1979)). The main change from Sciama (1990b) results from the recent establishment of a six times more stringent observational upper limit on F than we used there. Following Vogel et al (1995) we now adopt

$$F < 10^5 \text{photons cm}^{-2} \text{sec}^{-1} .$$

The contribution F_γ of cosmological decay photons to F is governed by the excess of E_γ over 13.6 eV, because the red shift eventually reduces the energy of a decay photon to below the ionisation potential of hydrogen. Writing

$$E_\gamma = 13.6 + \epsilon \text{ eV} ,$$

we have

$$F_\gamma = \frac{n_\nu}{\tau} \frac{c}{H_0} \frac{\epsilon}{13.6} ,$$

where τ is the decay lifetime, which in our theory satisfies $\tau < 3 \times 10^{23} \text{sec}$ (Sciama 1993). We thus require that

$$\epsilon < 0.39 h \text{ eV} .$$

Since ϵ contributes a relatively small fraction to E_γ , we here insert our final value for h of 0.55 (see below) to obtain

$$\epsilon < 0.21 \text{ eV} .$$

Hence we can write

$$E_\gamma = 13.7 \pm 0.1 \text{ eV} ,$$

and

$$m_\nu = 27.4 \pm 0.2 \text{ eV} ,$$

so that

$$\Omega_\nu h^2 = 0.293 \pm 0.003 .$$

(v) Ωh^2

To obtain Ωh^2 we combine $\Omega_\nu h^2$ and $\Omega_b h^2$ to obtain

$$\Omega h^2 = 0.300 \pm 0.003 .$$

Thus the density parameter of the universe Ωh^2 is determined with a precision of 1 per cent in the decaying neutrino theory.

3 THE DERIVATION OF H_0 AND t_0

To derive H_0 and t_0 from our value for Ωh^2 we proceed as follows. First we assume from the observations that

$$(a) h \geq 0.5$$

and

$$(b) t_0 \geq 12 \text{ Gyr}$$

(e.g. Chaboyer et al 1996). From (a) and $\Omega h^2 = 0.3$ we derive that $\Omega \leq 1.2$ and so (for a zero cosmological constant λ) that $t_0 \leq 13.4$ Gyr. From (b) and $\lambda = 0$ we derive that $\Omega \geq 1$ and $h \leq 0.55$. Combining these results we obtain

$$H_0 = 52.5 \pm 2.5 \text{ km. sec}^{-1} \text{Mpc}^{-1} ,$$

$$t_0 = 12.7 \pm 0.7 \text{ Gyr} ,$$

and

$$\Omega = 1.1 \pm 0.1 .$$

Since Ω is required to be so close to 1 it is tempting to assume that it is exactly 1 (or at least 1 to within the level of uncertainty to which we are working here). In that case the uncertainties in H_0 and t_0 are controlled by our one per cent uncertainty in Ωh^2 , and so we find that for $\lambda = 0$

$$H_0 = 54.8 \pm 0.3 \text{ km. sec}^{-1} \text{Mpc}^{-1} ,$$

and

$$t_0 = 11.96 \pm 0.06 \text{ Gyr} ,$$

(where 1 year is defined to be 3.15×10^7 seconds). It is gratifying that these precise predictions for the values of H_0, t_0 and Ω from the decaying neutrino theory are compatible with current observational estimates of these quantities.

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REFERENCES

- Alpher R.A., Follin J.W., Herman R.C., 1953, Phys.Rev. 92, 1347
 Bonifacio P., Molaro P., 1997, MNRAS, in press
 Chaboyer P., Demarque P., Kernan P.J., Krauss L.M., 1996, Science, 271, 957
 Fixsen D.J., Cheng E.S., Gales J.M., Mather J.C., Shafer R.A., Wright E.L., 1996, ApJ, 473, 576
 Gell-Mann M., Ramond P., Slansky R., 1979 in Proc. Supergravity Workshop eds. van Nieuwenhuizen P., Freedman D.Z., North-Holland, Amsterdam, p 315
 Hannestad S., Masden J., 1995, Phys. Rev.D, 52, 1764
 Hata N., Steigman G., Bludman S., Langacker P., 1997, Phys. Rev. D, 55, 540
 Izotov Y.I., Thuan T.X., Lipovetzsky V.A., 1997, ApJS, 108, 1
 Olive K.A., Skillman E., Steigman G., 1996, astro-ph 9611166
 Sciamia D.W., 1990a, ApJ, 364, 549
 Sciamia D.W., 1990b, Phys Rev Lett. 65, 2839
 Sciamia D.W., 1993, Modern Cosmology and the Dark Matter Problem (Cambridge University Press)
 Songaila A., Wampler E.J., Cowie L.L., 1997, Nat, 385, 137
 Vogel S.N., Weymann R., Rauch M., Hamilton T., 1995, ApJ, 441, 162
 Yanagida T., 1978, Prog Theor Phys, 135, 66

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